UNIVERSITI TEKNOLOGI MARA

SUFFICIENT CONDITION AND PARTIAL SUMS OF CERTAIN ANALYTIC FUNCTIONS

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Dissertation submitted in partial fulfillment
of the requirements for the degree of
Master of Science

Faculty of Computer and Mathematical Sciences

January 2014

AUTHOR'S DECLARATION

I declare that the work in this dissertation was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the results of my work, unless otherwise indicated or acknowledged as referenced work. This dissertation has not been submitted to any other academic institution or non-academic institution for any degree or qualification.

I, hereby, acknowledge that I have been supplied with the Academic Rules and Regulations for Post Graduate, Universiti Teknologi MARA, regulating the conduct of my study and research.

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Dissertation title	:	Sufficient Condition and Partial Sums of Certain Analytic Functions.
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Date	:	January 2014

ABSTRACT

In this study, we denote S as the class of normalized univalent functions in the unit disk $E = \{z : |z| < 1\}$ having the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots = z + \sum_{k=2}^{\infty} a_k z^k.$$

We also denote T as the class of functions with negative coefficients. An analytic and univalent function g is in T if it has the form

$$g(z) = z - a_2 z^2 - a_3 z^3 - \cdots = z - \sum_{k=2}^{\infty} a_k z^k.$$

We are interested in finding the sufficient condition for the class of δ-spiral-like functions of order α, $SP^*(\delta,\alpha)$ and for the class of δ-close-to-convex functions of order α, $G(\delta,\alpha)$ where

$$SP^*(\delta,\alpha) = \left\{ f \in S : \mathrm{Re}\left(e^{i\delta} \frac{zf'(z)}{f(z)} \right) > \alpha \right\} \text{ and } G(\delta,\alpha) = \left\{ f \in S : \mathrm{Re}\left(e^{i\delta} f'(z) \right) > \alpha \right\}$$

with $0 \le \alpha < 1$ and $|\delta| < \frac{\pi}{2}$. Moreover, we also considered necessary and sufficient conditions for the subclass of T that are δ-spiral-like functions of order α, $SPT^*(\delta,\alpha)$ and δ-close-to-convex functions of order α, $G^*(\delta,\alpha)$ satisfying the condition below

$$SPT^*(\delta,\alpha) = \left\{ g \in T : \mathrm{Re}\left(e^{i\delta} \frac{zg'(z)}{g(z)} \right) > \alpha \right\} \text{ and } G^*(\delta,\alpha) = \left\{ g \in T : \mathrm{Re}\left(e^{i\delta} g'(z) \right) > \alpha \right\}$$

with $0 \le \alpha < 1$ and $|\delta| < \frac{\pi}{2}$. In this study, we established the distortion theorem for the class $SPT^*(\delta,\alpha)$ and $G^*(\delta,\alpha)$ and evaluating the partial sums, $f_n(z)$ for the class $SP^*(\delta,\alpha)$. To get all those results, we follow through the work done by Silverman in 1975 and 1997. Moreover, we obtain sharp results of partial sums for each lower bound on $\mathrm{Re}\left\{ \frac{f(z)}{f_n(z)} \right\}$, $\mathrm{Re}\left\{ \frac{f_n(z)}{f(z)} \right\}$, $\mathrm{Re}\left\{ \frac{f'(z)}{f_n'(z)} \right\}$ and $\mathrm{Re}\left\{ \frac{f_n'(z)}{f'(z)} \right\}$.

ACKNOWLEDGEMENTS

IN THE NAME OF ALLAH THE MOST BENEFICIENT, THE MOST MERCIFUL.

First and foremost, I am grateful to Allah Al-Mighty for giving me the patience and strength to make it possible for me to accomplish this dissertation.

I would like to express my deepest gratitude to my supervisor Dr Shaharuddin Cik Soh from Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA for the continuous support and expert guidance in preparing a master dissertation. I appreciate all his contributions of ideas, time and invaluable insight pertaining to the completion of my dissertation.

Special thanks and appreciation to Universiti Teknologi MARA for providing me the scholarship. Thank you for having faith in me and giving me the opportunity in reaching my educational goal.

I like to thank my parents for their undivided support and interest who inspired me and encouraged me to go on my way. My thanks go to my loved ones, husband and children for cheering me up and stood by me throughout the good times and bad.

At last but not the least, I want to thank my friends who appreciated me for my work and motivated me throughout my studies.

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